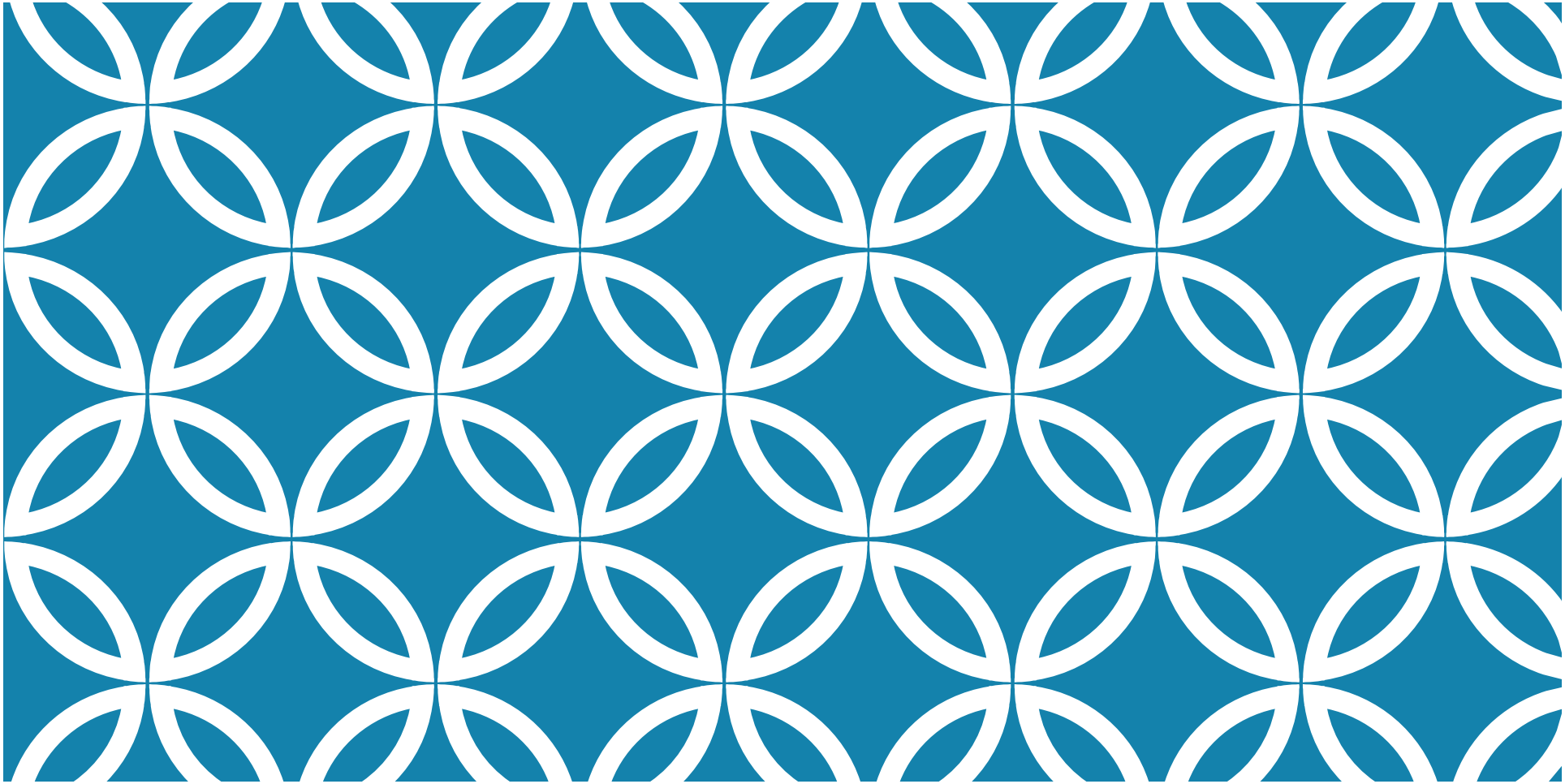

SHIFTING PARADIGM IN PROBLEM SOLVING: ALGEBRAIC VERSUS ARITHMETIC THINKING

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OUTLINE OF THE SESSIONS

Session 1:

Creation of an awareness of the limits of the operational paradigm

Session 2 :

Introduction of the relational paradigm in relation to algebraic and arithmetic thinking

Session 3:

Theorization of the two paradigms in problem solving; algebraic versus arithmetic thinking

ACTIVITY 1: DISCUSSION

Why should we solve arithmetic word problems?

Maria and her mom wanted to plant 38 tulips in front of their house. Working for 25 minutes they already planted some tulips. There are 17 tulips more to plant to finish their project. How many tulips did they plant?

Maria and her mom wanted to plant A tulips in front of their house. Working for B minutes they already planted some tulips. There are C tulips more to plant to finish their project. How many tulips did they plant?

WHY SHOULD WE SOLVE ARITHMETIC WORD PROBLEMS?

Reasons (first problem)	Reasons (second problem)
Game	To generalize important information
to practice a concept in a context	Need to see the question; what is needed to solve the problem; Imply A>C
Simulate what we see in a daily life	Problem structure
Mathematize a situation	Engagement of students is problematic
Time orientation	Engagement in mathematical thinking

HOW DO WE UNDERSTAND A WORD PROBLEM?

Pierre had 8 marbles. Then he won 5 other marbles. How many marbles does he have now?

As a story?

Pierre had 13 marbles. Then he lost some marbles. Now he has 8 marbles. How many marbles has he lost?

As an operation?

Pierre had 8 marbles. Then he won some marbles. Now he has 13 marbles. How many marbles has he won?

As a relationship?

ACTIVITY 2
THE CAPTAIN'S GAME

Choose a captain in your team.

The captain gets out of the room!

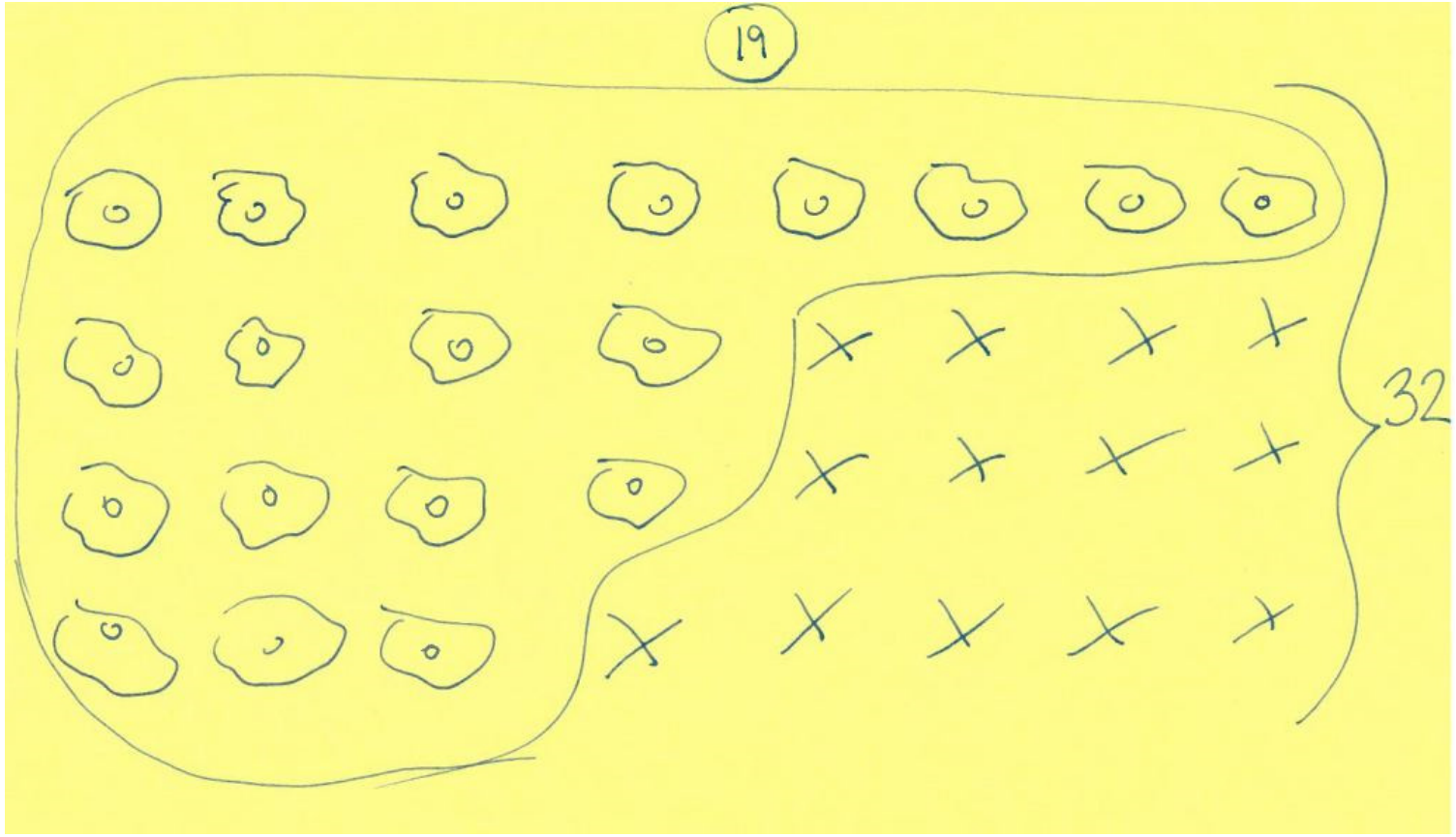
The sailors must represent the problem <u>without</u> :

- words or letters
- symbols of mathematical operations
- numbers except those from the problem's text

They can use : =, ?, ->, …

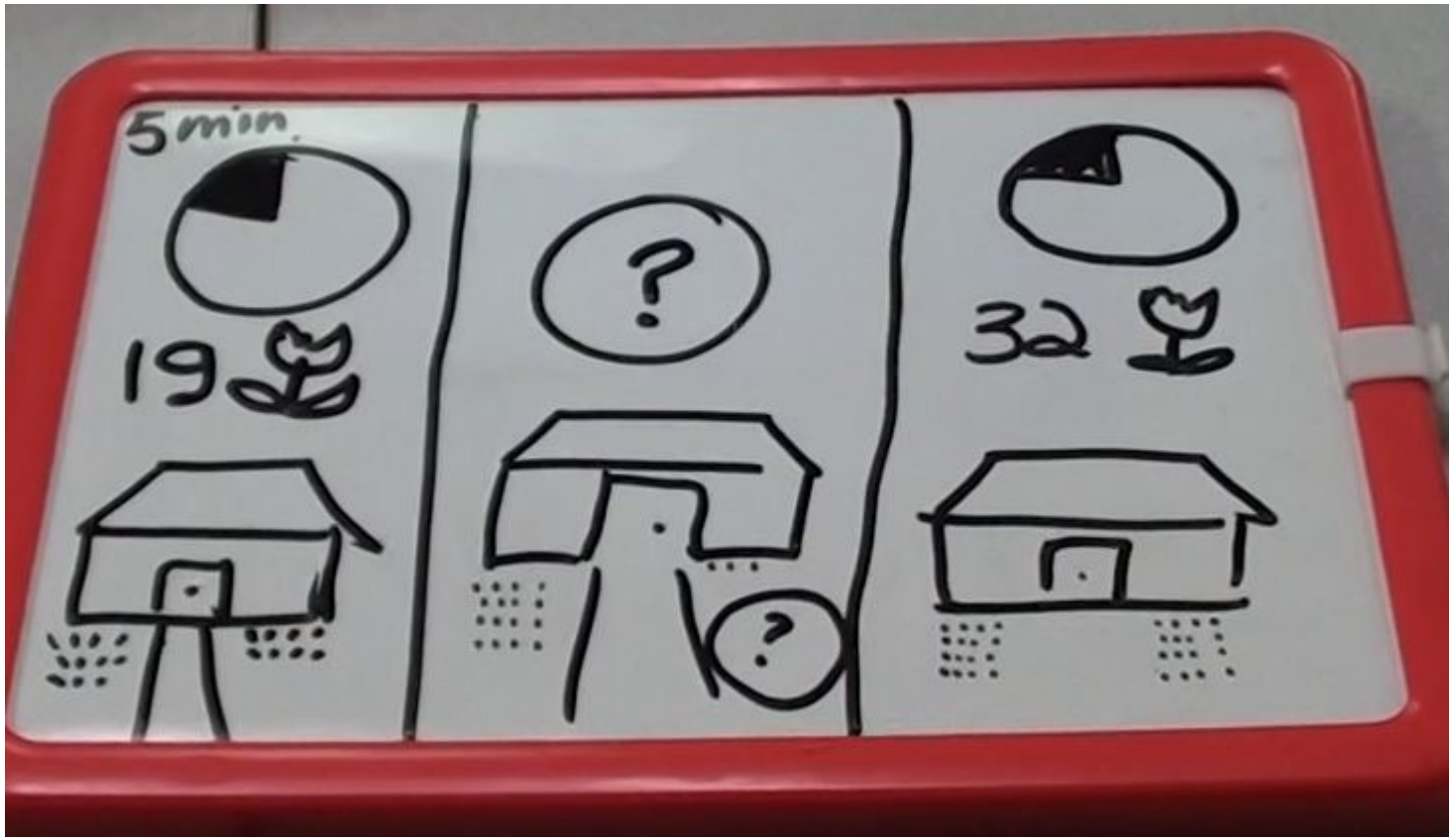
The captain should interpret the representation and calculate the answer

TULIPS PROBLEM

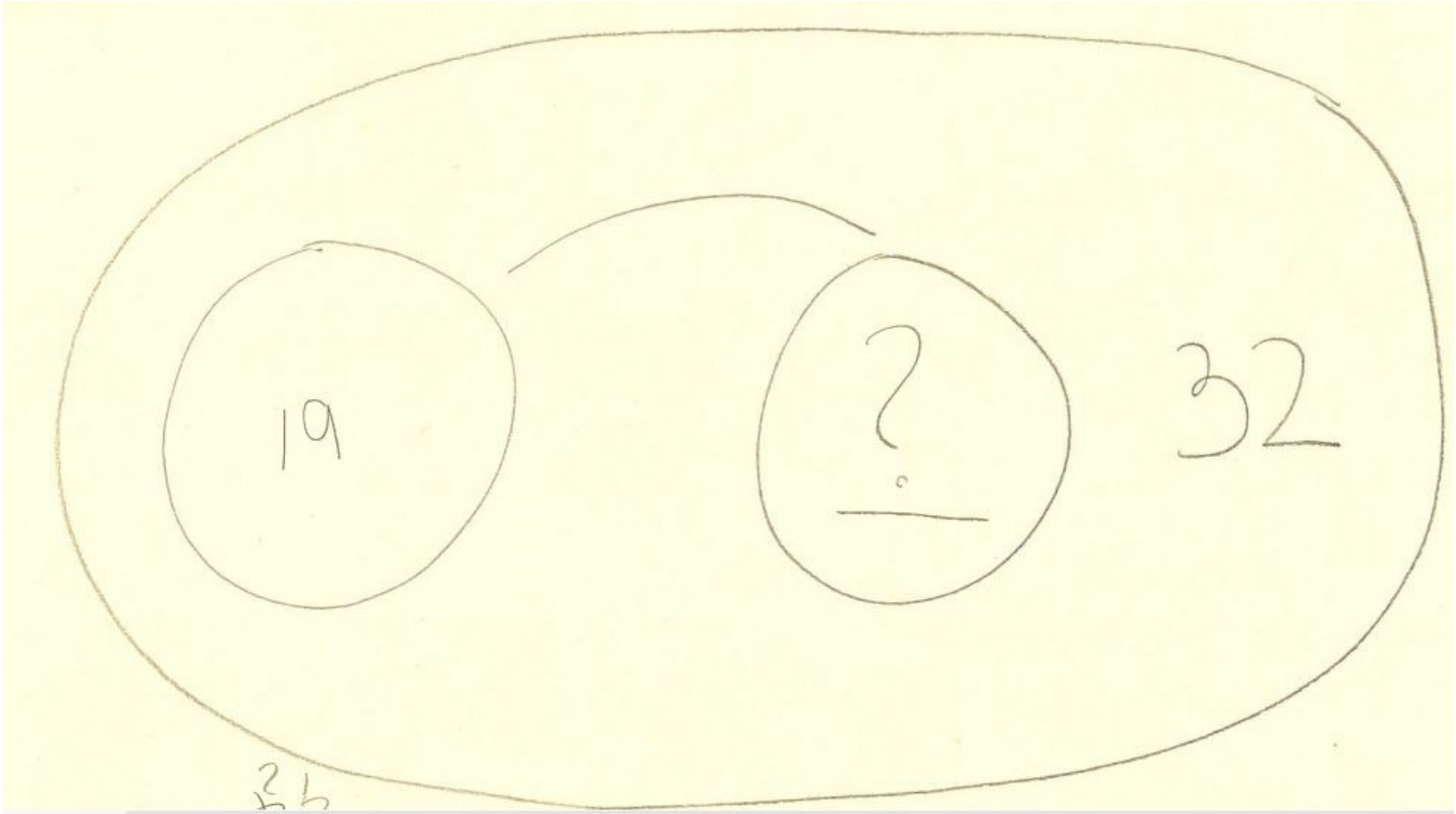


Tulips problem

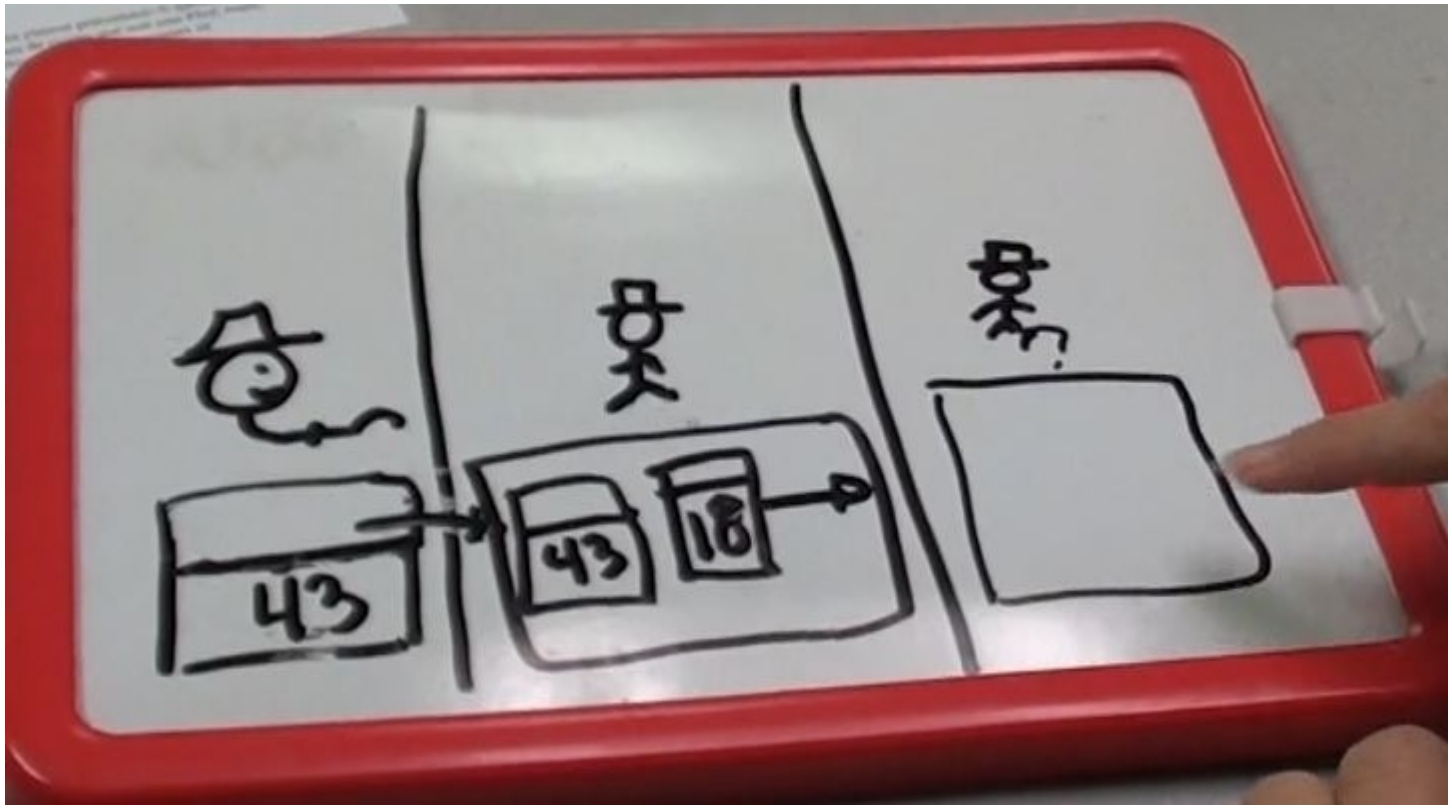
Tulips problem



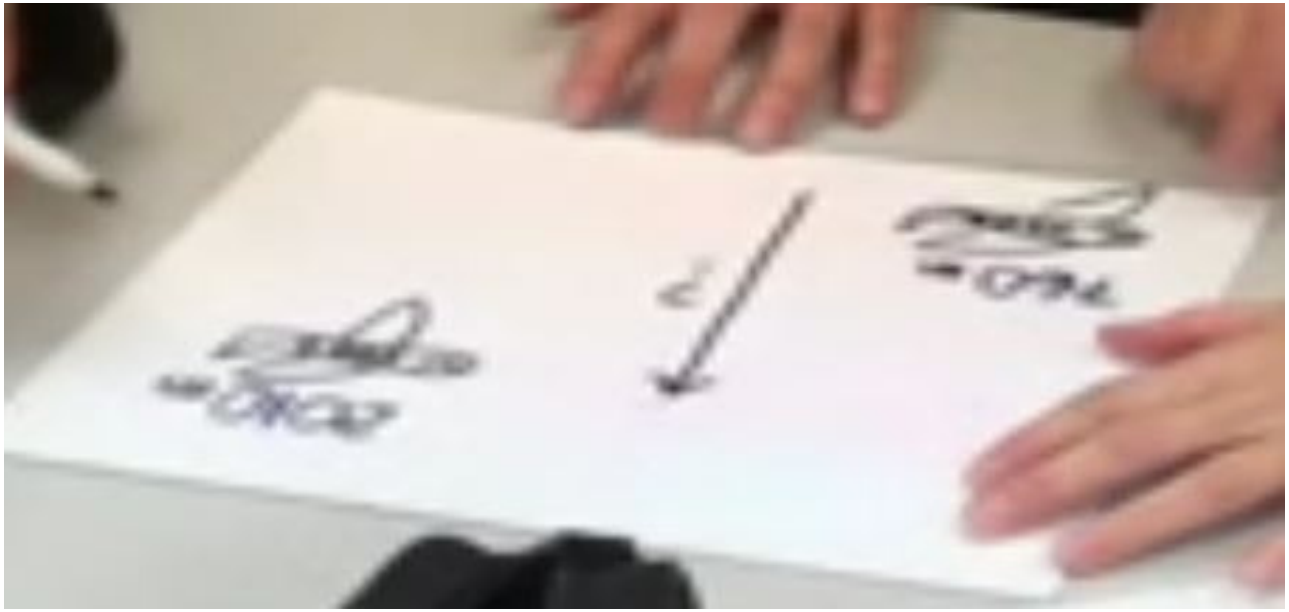
Tulips problem



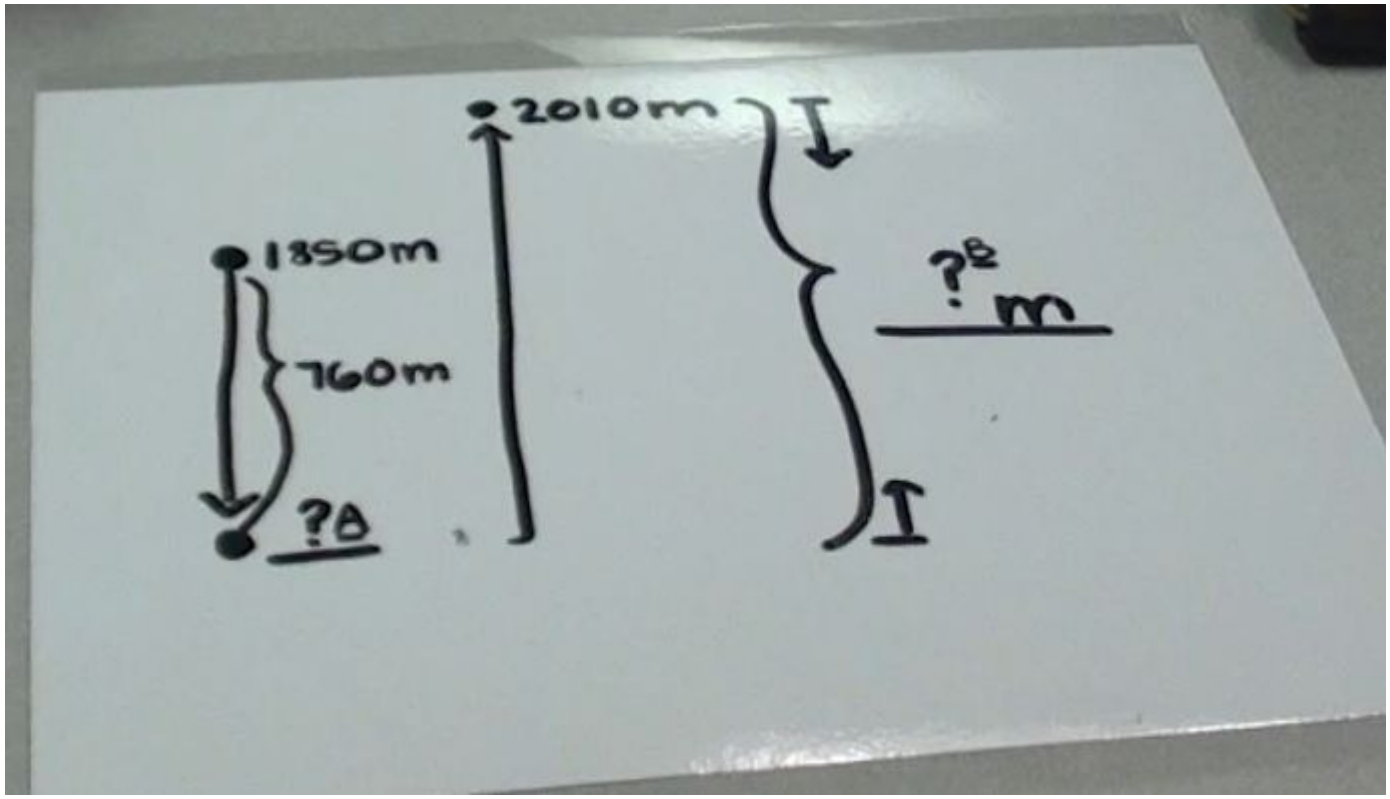
Pirates problem



Airplane problem

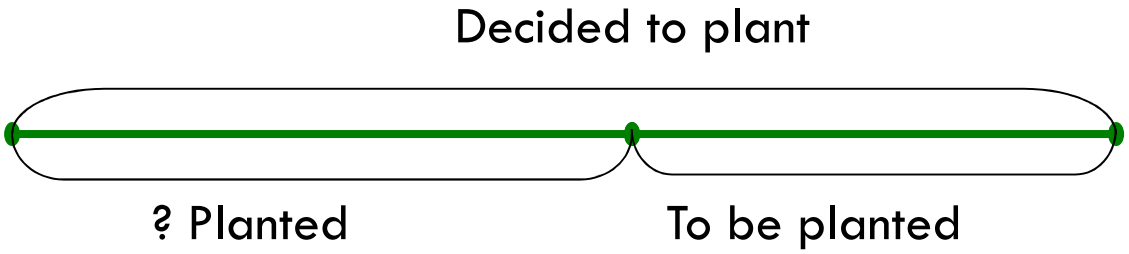


Airplane problem



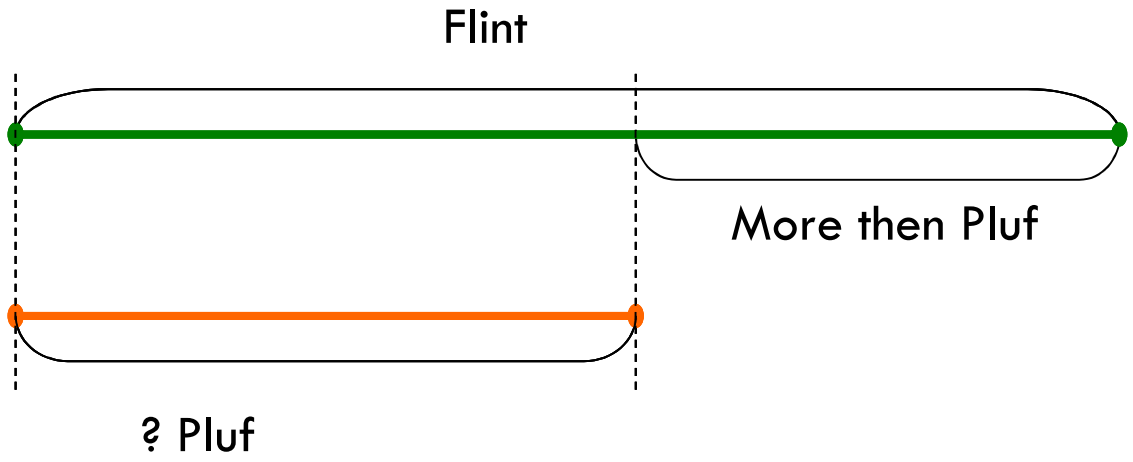
Tulips problem

Annie helps her mother to plant tulips near the house. They decided to plant 32 flowers. They have been working for 5 minutes and have planted a number of flowers. There are now 19 flowers to be planted. How many flowers did they plant?



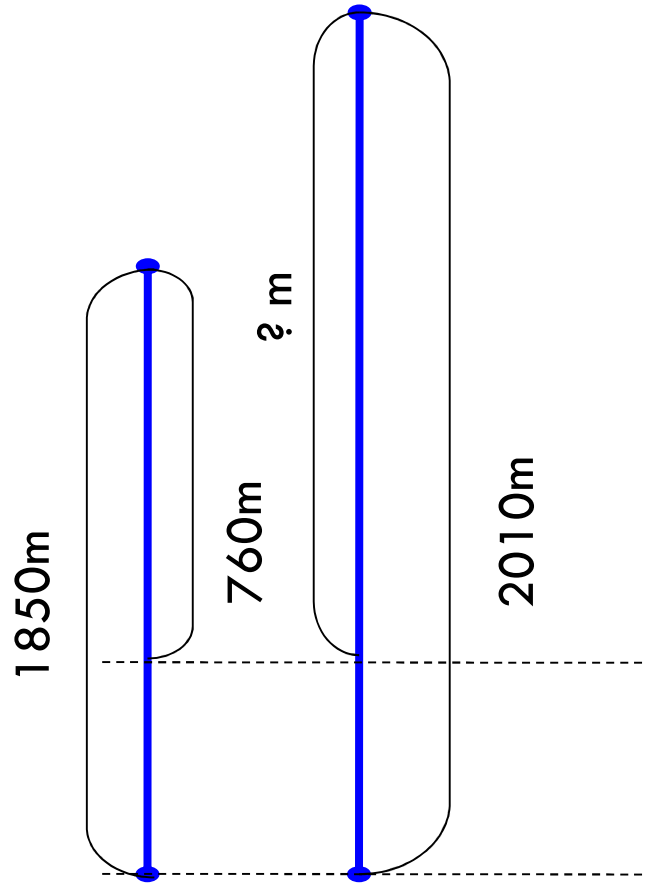
Pirates problem

In his treasure chest, pirate Flint has 43 diamonds. He has 18 diamonds more than his friend Pluf. But, except you, nobody knows it. Can you tell how many diamonds are there in the Pluf's treasure chest without having it opened?

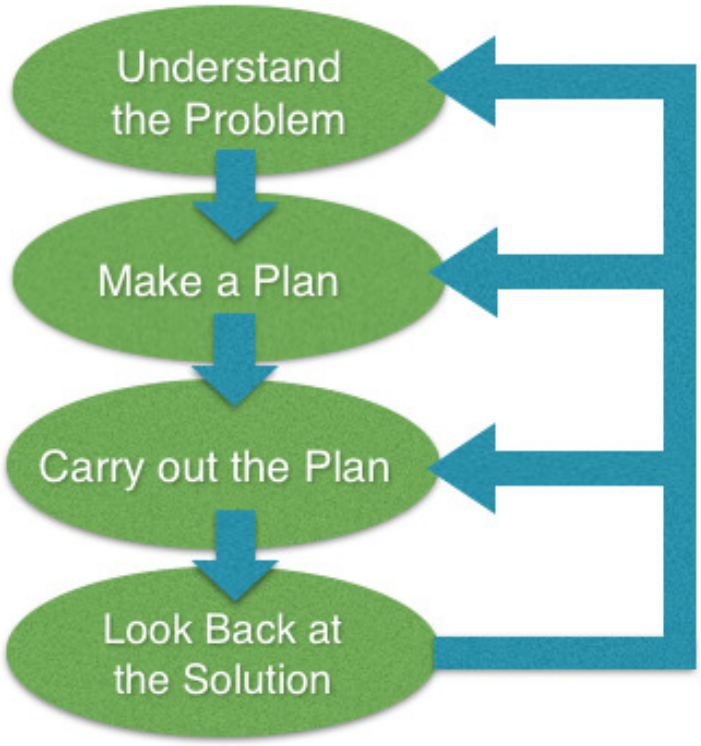


Airplane problem

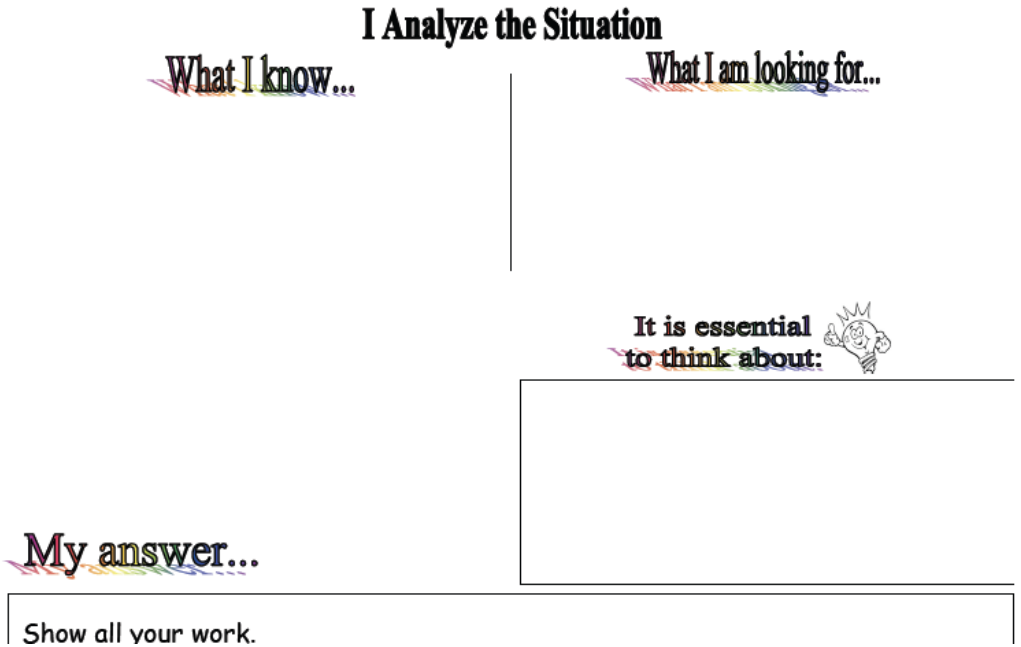
A plane was flying at the altitude of 1850m. To take some pictures of the Earth, it went down by 760m. Then it went up to the altitude of 2010m. By how many meters did the plane went up after taking pictures?



HOW DO TEACHERS TEACH PROBLEM SOLVING STRATEGIES?



Polya's Problem Solving Model



HOW DO TEACHERS TEACH PROBLEM SOLVING STRATEGIES?

Grade I class
Beginning of the school year
The teacher would like to see what her students can do to solve a problem

Students' solutions:
Drew butterflies without reference to the mathematics in the problem;

Students' solutions:

Drew the correct number of butterflies, but without any particular organisation;

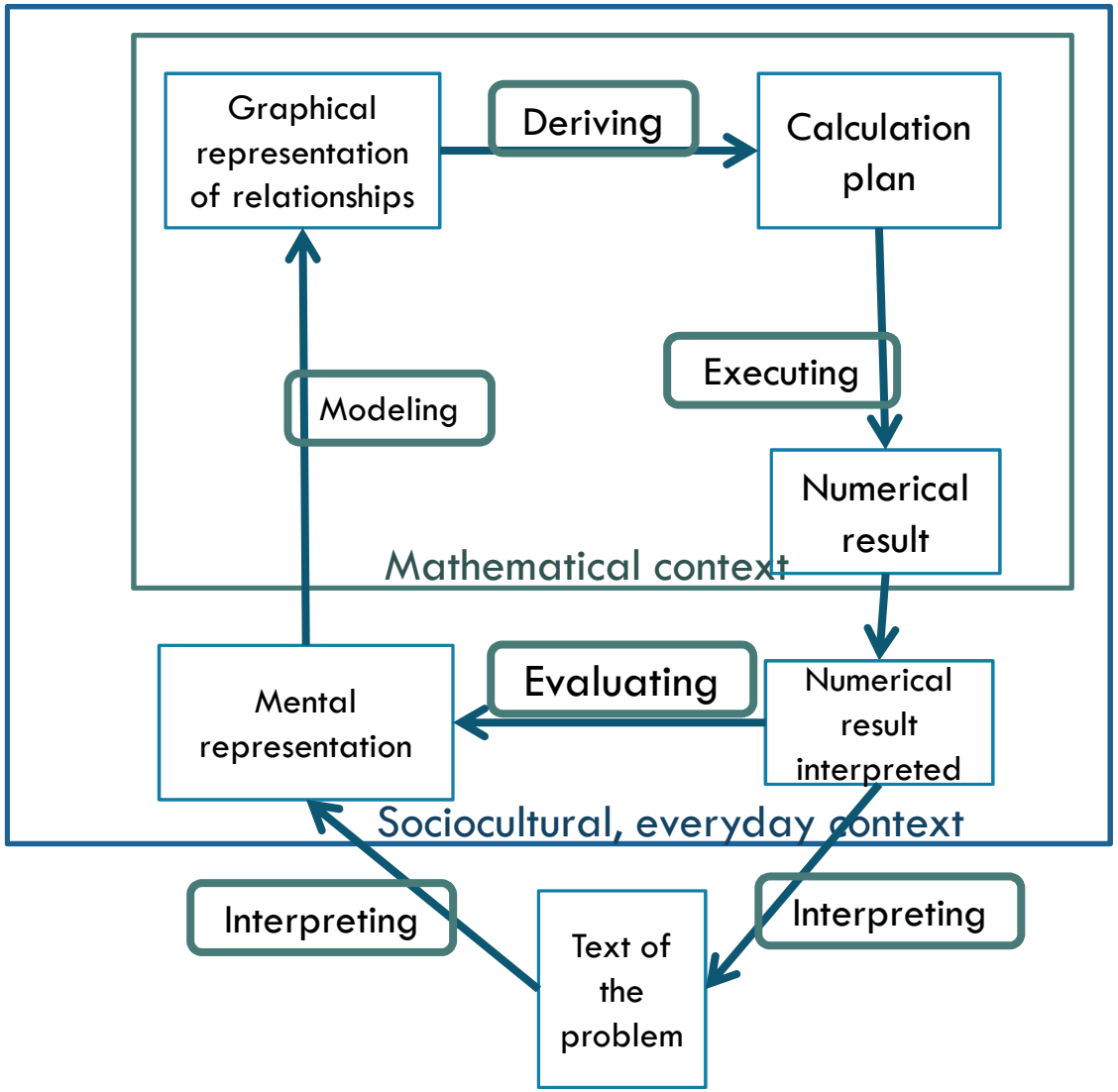
Students' solutions:

Circled the numbers in the problem's text, but didn't know how to find the difference;

Students' solutions:

- Used a number line to find the difference between the numbers;
- Wrote down a correct mathematical expression and mentally calculated the answer.

ETHNO-MATHEMATICAL MODEL



(Mukhopadhyay & Greer, 2001; Polotskaia, 2015; Savard, 2008)

WHAT DID WE LEARN SO FAR?