

**SHIFTING PARADIGM IN  
PROBLEM SOLVING: ALGEBRAIC  
VERSUS ARITHMETIC THINKING**

**Annie Savard**  
McGill University

**Elena Polotskaia**  
Université du Québec en  
Outaouais

# OUTLINE OF THE SESSIONS



## Session 1:

Creation of an awareness of the limits of the operational paradigm

## Session 2 :

Introduction of the relational paradigm in relation to algebraic and arithmetic thinking

## Session 3:

Theorization of the two paradigms in problem solving; algebraic versus arithmetic thinking

# Welcome!

## Practical goals

- a game - imagining things

Combining

open-ended questions  
combining  
open-ended questions  
combining  
open-ended questions  
combining  
open-ended questions  
combining

- Practice a concept they learn in a context

- Simulate what they see in their daily life

- Mathematize a Situation

- Time situation

- to generalize important information

- need to see the question

- what is needed to solve

- Conditional ( $^{\circ}A$  is bigger than  $^{\circ}C$ )

- problem structure

Situation?

skill or problem?

More mathematical

build self-confidence

engagement for students?

engagement in mathematical thinking

# REVIEW OF SESSION I



Reasons (first problem)	Reasons (second problem)
Game	To generalize important information
to practice a concept in a context	Need to see the question; what is needed to solve the problem; Imply $A > C$
Simulate what we see in a daily life	Problem structure
Mathematize a situation	Engagement of students is problematic
Time orientation	Engagement in mathematical thinking

# HOW DO TEACHERS TEACH PROBLEM SOLVING STRATEGIES?

Grade I class

Beginning of the school  
year

The teacher would like  
to see what her students  
can do to solve a  
problem

## THE PROBLEM

Yesterday, students from two classes flew butterflies out. There were 13 butterflies in the aquarium in the class of Ms. Guylaine. There were 7 butterflies in the aquarium of the class of Ms. Sara. In what class there were more butterflies? How many more?



Students' solutions:

Drew butterflies without reference to the mathematics in the problem;

Students' solutions:

Drew the correct number of butterflies, but without any particular organisation;







Students' solutions:

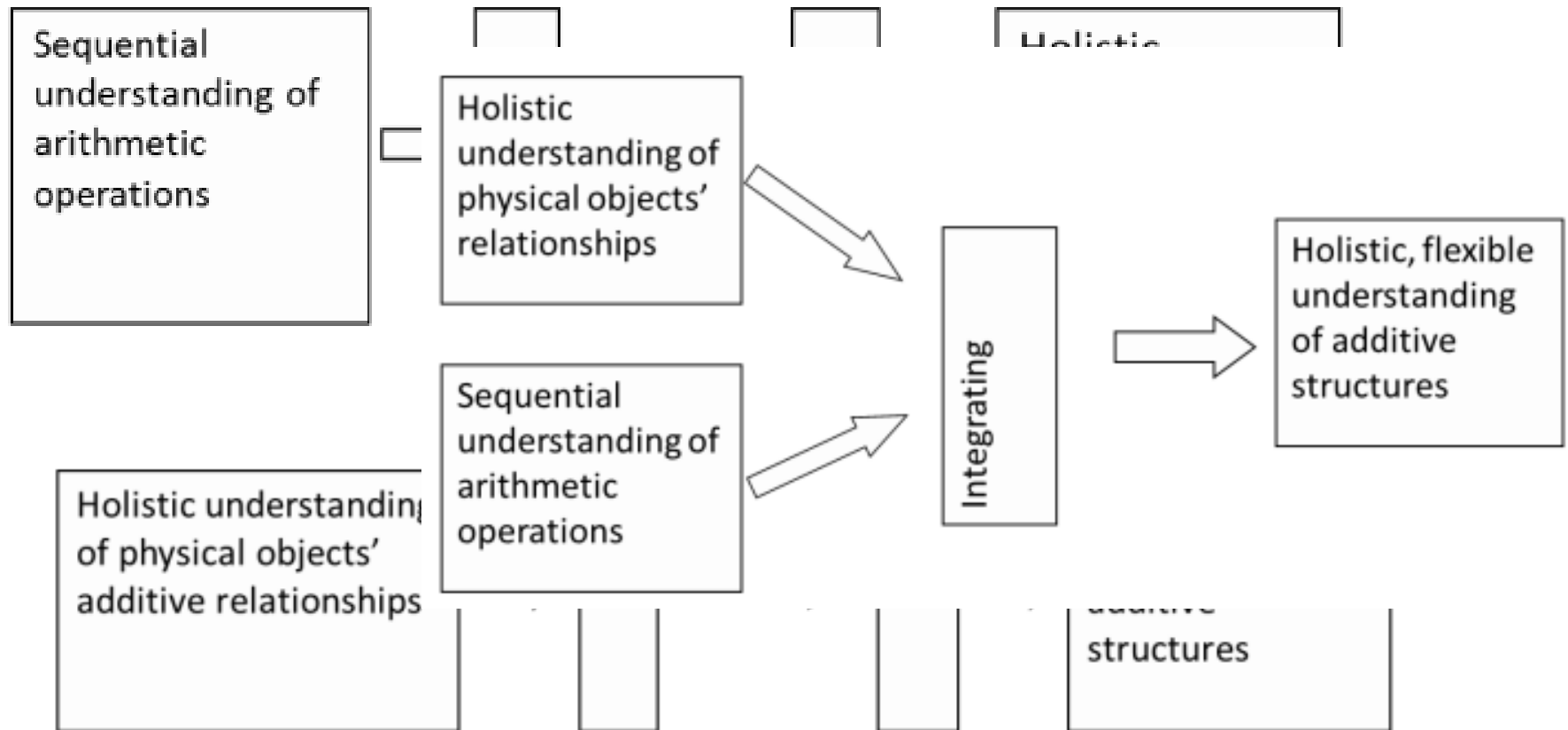
Circled the numbers in the problem's text, but didn't know how to find the difference;

Students' solutions:

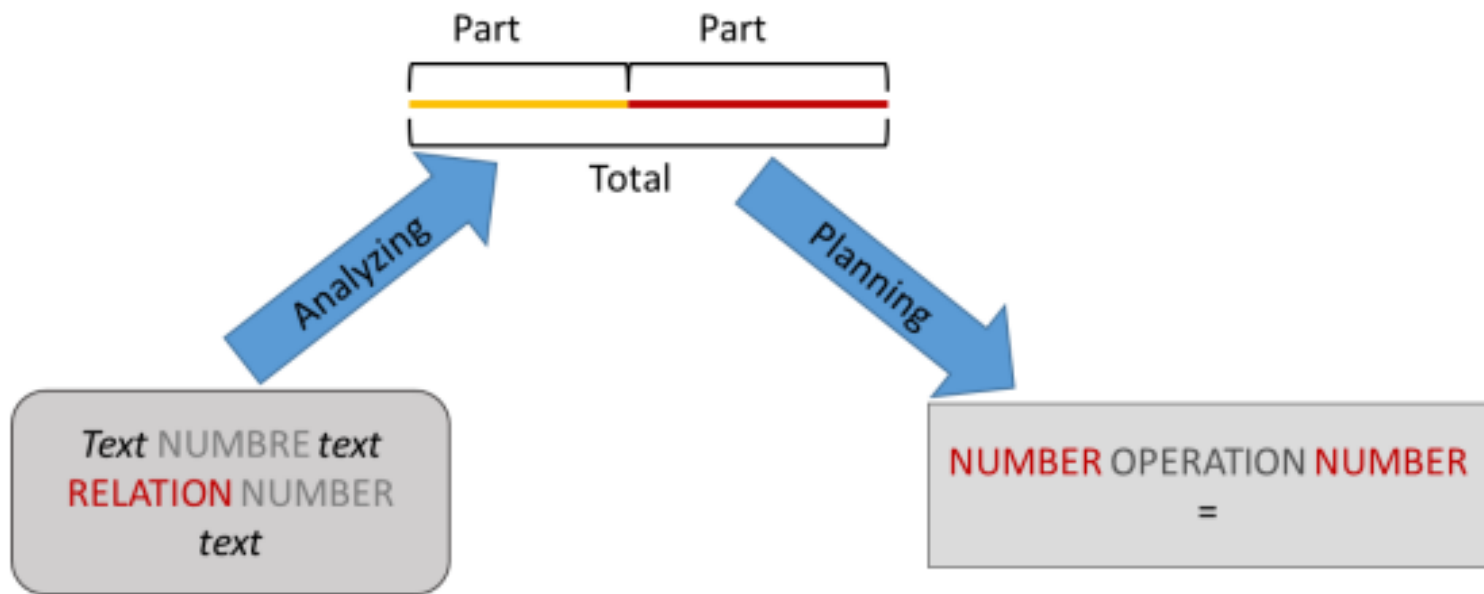
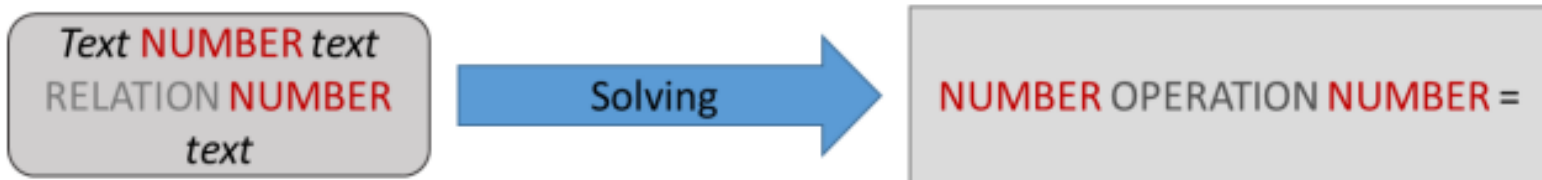
Used a number line to find the difference between the numbers;

Wrote down a correct mathematical expression and mentally calculated the answer.

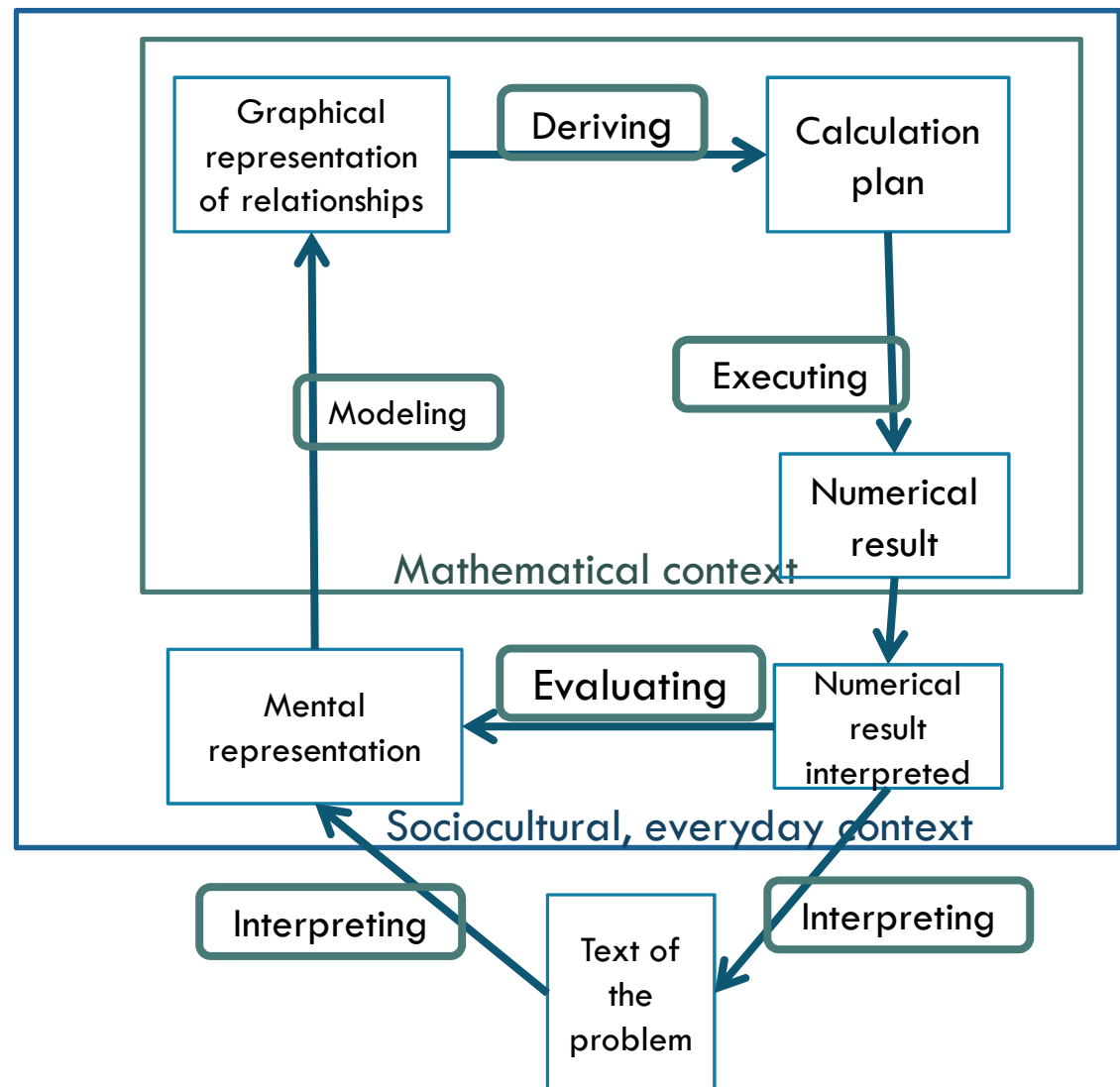
# DEVELOPMENTAL PERSPECTIVE



# SOLVING PERSPECTIVE



# ETHNO-MATHEMATICAL MODEL



(Mukhopadhyay & Greer, 2001;  
Polotskaia, 2015; Savard, 2008)

# WHERE DID WE START CONSTRUCTING RELATIONAL THINKING?



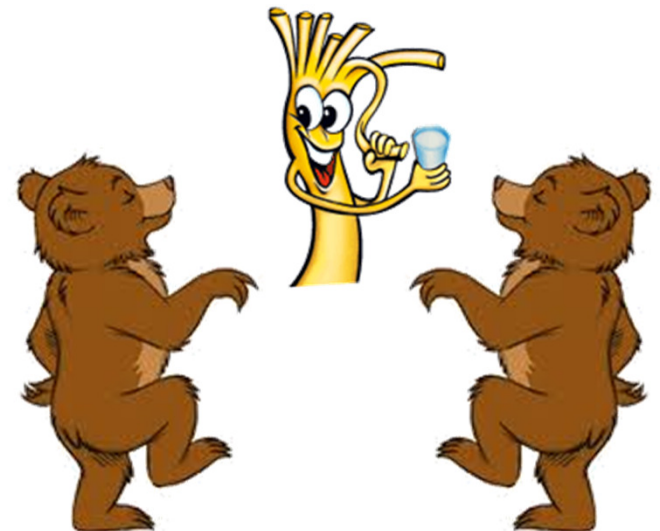
Fairness of sharing and a comparison strategy for continuous quantities

Developing a **rigorous** comparison strategy

*Compared-Compared-Difference* relationship and structure.

Reconstruction of missing part

*Part-Part-Whole* relationship and structure



# THE STORY OF THE TWO CUBS



Once upon a time, two little bear cubs found a cheese stick. It is possible that this cheese stick had fallen from the picnic basket of the family who had visited the forest this morning. The cubs are very hungry. They rush towards it and grab the cheese stick at the two opposite ends:

*“I’m the one who found it!”* says the first little bear.

*“No! It’s me! It’s my cheese stick!”* cried the second



# THE STORY OF THE TWO CUBS



They shout and shout, and pull so hard on the cheese stick that it breaks into two pieces!

*“Your piece is bigger than mine!”* yells out the first cub

*“No! That’s not true! Yours is bigger!”* says the second little bear.



The two cubs start crying and screaming again. They shout and shout until an old fox comes by to see who was making so much noise.



# THE STORY OF THE TWO CUBS



*“What’s happening here?” says the Old Fox.  
“Why are you fighting like this and why are you making all this noise?”*

The two cubs explain to the old fox that they wish to share the cheese, but that they cannot reach an agreement.



*“Very well” says the Old Fox, “I will help you”.*

# THE STORY OF THE TWO CUBS



The Old Fox takes the two pieces of cheese, looks at them, and then takes a little bite from one of them. He looks at the pieces again and shakes his head, unsatisfied. He eats another little bite, but this time from the other piece. Once again, the two pieces of cheese are different. The fox then repeats the procedure, again and again and again until the two pieces become very, very small. Satisfied and licking his lips, the Old Fox gives back the tiny bits of cheese to the Two Cubs.



*“Here are your two pieces of cheese. Now, they are equal!”* says the Old Fox.

And he happily returns to the forest - with a full belly.



# VIDEO FROM CLASSROOM

Bear cubs 2. Students constructing a solution for missing part.

*Is there any link  
to problem-  
solving?*

# BEFORE SOLVING A PROBLEM



*Three friends discuss their age :*

*Marta : My cousin Stephane is 12 years older than me.*

*Camille : I am 10 younger than Stephane.*

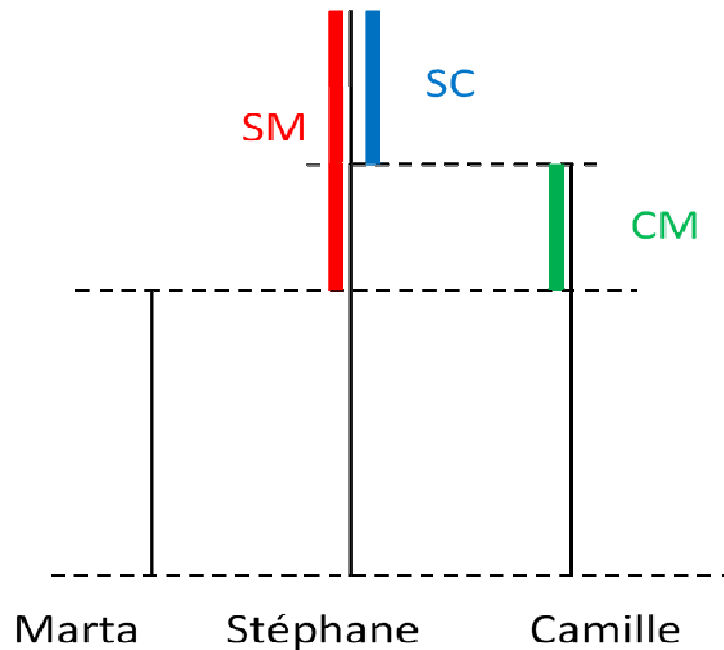
*Thomas : You, Camille, you is 4 years older than Marta.*

*What can you say about this story?*

*Can you represent the situation by a segment diagram?*

*What did you think about in order to represent the situation?*

# CONSTRUING OPERATIONS



$$SM = SC + CM$$

$$SC = SM - CM$$

$$CM = SM - SC$$

Hypothesis 1 : Marta is wrong (number 12 is incorrect)

Hypothesis 2 : Thomas is wrong (number 4 is incorrect)

Hypothesis 3 : Camille is wrong (number 10 is incorrect)

*What did we work on?*

- 1. Story (context)*
- 2. Operation (arithmetic)*
- 3. Relationships (algebraic)*



# VIDEO FROM CLASSROOM

Cinnamon hearts

*Is there any  
link to  
problem-  
solving?*

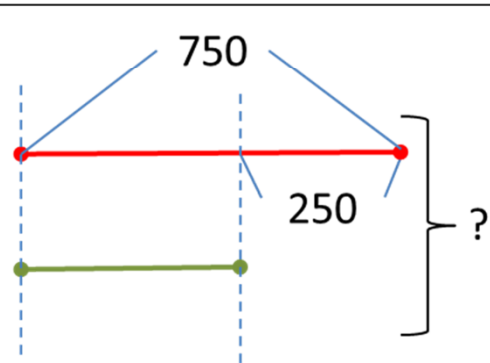
*Working in a team, try to solve the enigma.*

*Share your emotional and cognitive experience with the group.*

# ACTIVITY 1

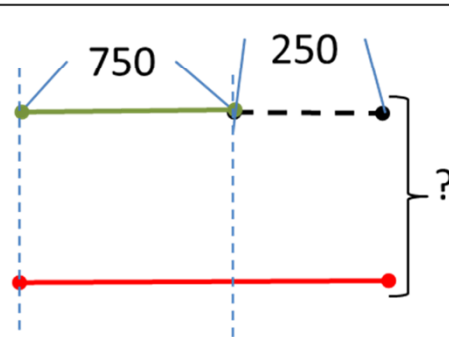
1. The grandmother knits wool sweaters. For dad's sweater, she used 250g of wool more than for mom's sweater. It took her 125 days to knit both sweaters. Both sweaters weigh 750g together. How much does dad's sweater weigh?

a



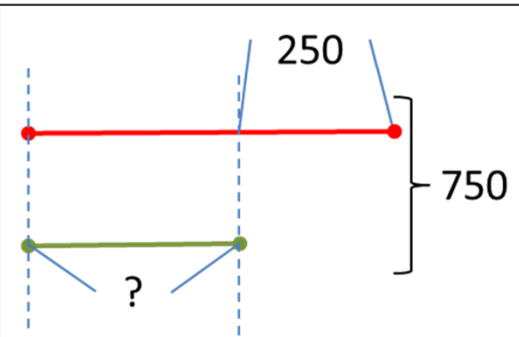
2. Grandfather is a beekeeper. He has collected 250g of honey less from the first hive than from the second. The first hive gave him 750g of honey. The grandfather put all the honey in 125 small pots. How much honey did he put in all pots?

b



3. My father is a writer. He wrote a great novel consisting of three parts. The first part of this novel contains 250 pages more than the second one. The third part is 125 pages shorter than the second one. The first part contains 750 pages. How many pages does the novel contain?

c





*What did we work on?*

*1. Story (context)*

*2. Operation (arithmetic)*

*3. Relationships (algebraic)*





# VIDEO FROM CLASSROOM

Enigmas discussion

*Is there any link to  
problem-solving?*

## ACTIVITY 2

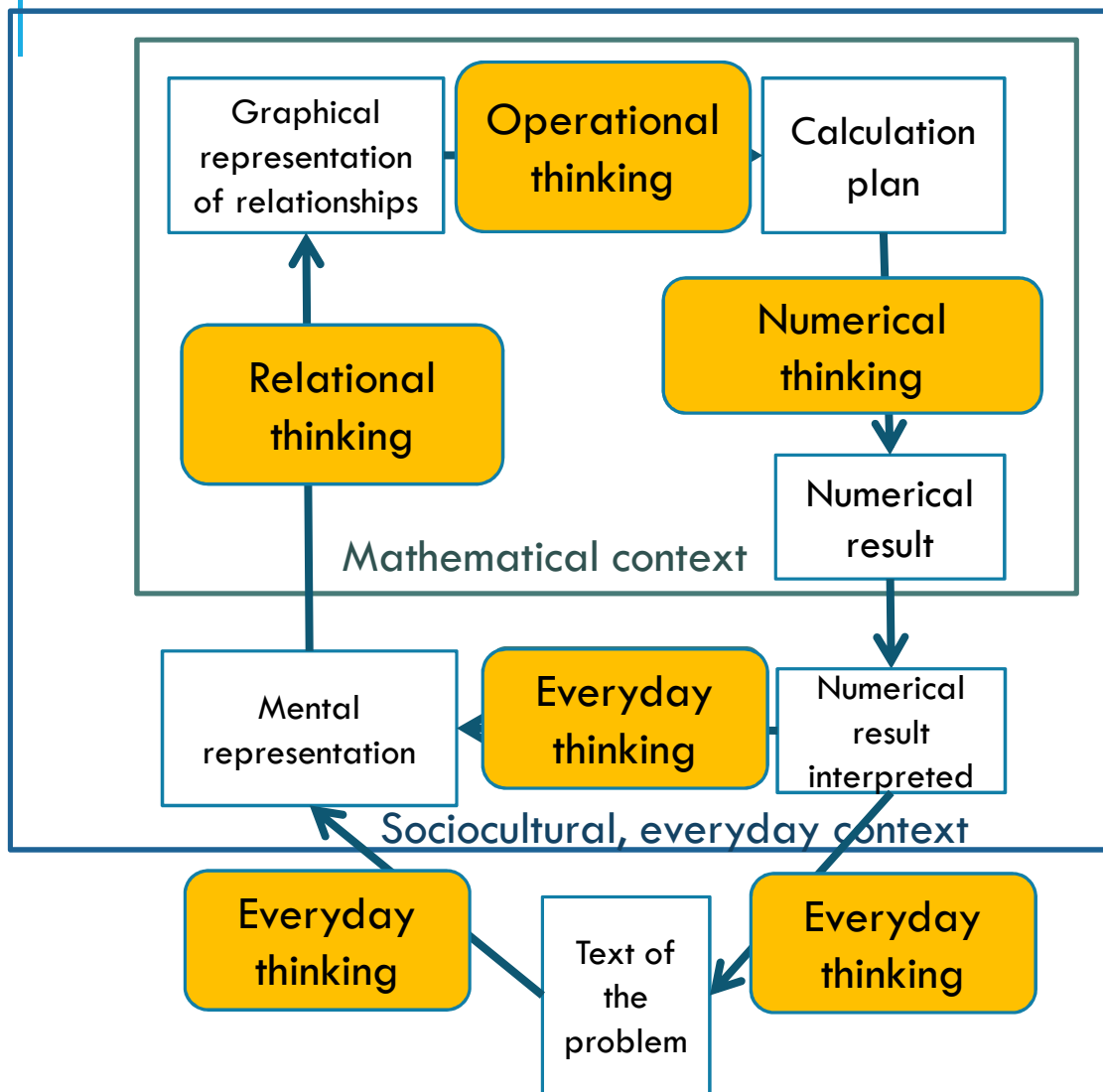


1. *Working in a team*
2. *Pick a problem*
3. *Try to solve while going through the problem-solving cycle (analyze, represent by a segment diagram, solve)*
4. *Share your emotional and cognitive experience with the group.*

# PROBLEMS

1. There was the same number of books standing in two shelves. The librarian moved 38 books from the first shelf to the second. How many more books are there now in the second shelf compared to the first?
2. There were twice as many books in the first shelf than in the second. Nicolas counted seven more books in the third shelf than in the second. How many books were there in the first shelf if in all three shelves there were 51 books?
3. A bookstore received 750 copies of a novel. On the First day a number of copies were sold. The next day, half of the number of copies sold on the first day was sold. The bookstore has 327 copies of the novel left . How many copies were sold the first day?
4. Asterix and Obelix compete by force. One of their tests is that of push-ups. Together they made 36 push-ups, but Obelix did 14 less than Astérix. How many push-ups each of them did?
5. Mother has prepared a pitcher of milk (4 glasses) for her three children. Frideric took a glass and a half. Kiril took three quarters of a glass. Leon took milk as well. There still is a glass of milk in the pitcher. How much did Leon drink?
6. Damir and Cassim are wool sheep breeders. Damir owns 573 sheep. Cassim has 159 more sheep than Damir. Today, Damir and Cassim worked together to mow (cut the wool of) their sheep. They mowed two thirds of all the sheep. How many sheep still keep their winter coats?
7. The golf course has a rectangular shape and it is fenced. After a reorganization, the width of the land was increased by 0.54 km and its length was reduced by 0.3 km. Is the length of the fence changed? Of how much?

# REASONING DEVELOPMENT



What kind of thinking is promoted through the problem-solving process?



# CONCLUSIONS OF SESSION II