

Multiplicative relationships and their visual representations

Relations multiplicatives et leurs représentations visuelles

Word problems can be defined as verbal descriptions of problem situations wherein one or more questions are raised, the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement. [...] Typically a student who is confronted with the problem in the context of a mathematics lesson or a mathematics test is required to give a numerical answer to a specific question by making explicit and exclusive use of **the quantities given in the text and mathematical relationships between those quantities** inferred from the text. (Verschaffel, Greer, & De Corte, 2000, p. ix)

In his seminal work, Davydov (1982) put forward the idea of **quantitative relationships as mathematical concepts** that we need to teach and learn in elementary school, even prior to numbers. He argues that the concept of number appears from the **multiplicative comparison of two magnitudes (or quantities), one playing the role of unit of measurement and the other being measured.**

We make a clear **distinction** between the relationship governing a situation and a solution strategy for a particular problem based on the situation. **We suggest that a solution strategy should be derived from the relationship and not from the particularity of the problem.** (example: the train story presents a grouping situation, but if the number of wagons is to be found, the operation to use would be division and not multiplication)

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Mathematically Impossible Situation

Il y a 24 passagers dans le train.
Le train est composé de 12 voitures.
Dans chaque voiture, il y a 6 passagers.



The multiplicative relationship between three values is corrupted.

There are 24 passengers in the train. The train has 12 wagons. There are 6 passengers in each wagon.

Davydov (1982) describes the concept of quantitative relationship as a low “by which **the relation between two elements determines a unique third element as a function**” (p. 229)

Terminology used in school and literature	Problem	Representation	Relationship according to Davydov theory	Representation																				
Repeated addition Addition répétée	Gustave receives 3 objects each day. How many objects will he have in 4 days? Gustave reçoit 3 objets par jour pendant 4 jours. Combien d'objets a-t-il reçus?		Total quantity measured = 12 objects Unit = 3 objects per day Number of repetitions (or coefficient) = 4 Unit of reference = 1 object	Quantité totale = 12 objets Unité = 3 objets Nombre de répétitions (coefficient) = 4 Unité de base = 1 objet																				
Sharing or partition Partage	Gustave share 12 objects between 4 people. How many objects each person will have? Gustave partage 12 objets entre 4 personnes. Combien d'objets chaque personne doit-elle recevoir?		Total quantity measured = 12 objects Unit = ? objects per person Number of people (coefficient) = 4 Unit of reference = 1 object																					
Measurement or quotient Contenance	Gustave put 12 objects in bags, 3 objects per bag. How many bags will it make? Gustave met 12 objets dans des sacs, 3 objets par sac. Combien de sacs va-t-il utiliser?		Total quantity measured = 12 objects Unit = 3 objects per bag Number of bags (coefficient) = ? Unit of reference = 1 object																					
Comparison Comparaison	Gustave has 3 objects. Maude has 4 times more than Gustave. How many does she have? Gustave a 3 objets. Maude a 4 fois plus d'objets que Gustave. Combien d'objets Maude a-t-elle?		Total quantity measured = ? objects Unit of measurement = 3 objects Coefficient = 4 times more Unit of reference = 1 object																					
Rectangular disposition Disposition rectangulaire	In the classroom, there are 4 rows of 3 desks each. How many desks are there in the classroom? Dans la classe, il y a 4 rangées de 3 pupitres chaque. Combien de pupitres y a-t-il dans la classe?		Total quantity = 12 desks Dimension 1 = 3 rows Dimension 2 = 4 desks per row Unit of reference = 1 desk	Total quantity = 12 Desks Unit = 3 desks per row Number of rows (coefficient) = 4 Unit of reference = 1 desk																				
Cartesian product or area Produit cartésien ou l'aire	Gustave has 3 pants and 4 shirts. How many different outfits can he make? Gustave a 3 pantalons et 4 chemises. Combien de costumes différents (pantalon-chemise) peut-il faire?	Table <table border="1"><tr><td></td><td>P1</td><td>P2</td><td>P3</td></tr><tr><td>S1</td><td>P1S1</td><td>P2S1</td><td>P3S1</td></tr><tr><td>S2</td><td>P1S2</td><td>P2S2</td><td>P3S2</td></tr><tr><td>S3</td><td>P1S3</td><td>P2S3</td><td>P3S3</td></tr><tr><td>S4</td><td>P1S4</td><td>P2S4</td><td>P3S4</td></tr></table> Tree 		P1	P2	P3	S1	P1S1	P2S1	P3S1	S2	P1S2	P2S2	P3S2	S3	P1S3	P2S3	P3S3	S4	P1S4	P2S4	P3S4	Total quantity = 12 outfits Dimension 1 = 3 pants Dimension 2 = 4 shirts Unit of reference = 1 outfit	
	P1	P2	P3																					
S1	P1S1	P2S1	P3S1																					
S2	P1S2	P2S2	P3S2																					
S3	P1S3	P2S3	P3S3																					
S4	P1S4	P2S4	P3S4																					

Notre salle de concert peut accueillir 1200 spectateurs. Il est composé de trois zones: rouge, verte et bleue. Il y a 254 sièges dans la zone rouge ce qui est deux fois moins que le nombre de sièges dans la zone verte. Combien y a-t-il de sièges dans la zone bleue?

Secondaire 1
Adaptation scolaire

Notre entreprise familiale produit des muffins pour toute la ville. Nous avons 5 fours pour les cuire. On utilise des plateaux de 4 rangées de 6 muffins. Chaque four cuit 5 plateaux à la fois. Il faut 30 minutes pour faire cuire tous ces muffins. Les 5 fours fonctionnent pendant 5 heures chaque jour. Aujourd'hui, nous avons produit 3600 muffins seulement parce que certains fours ont été en panne toute la journée. Combien de fours ont été en panne?

Primaire 6^e
Régulier

Maxime participe à la course à relais. Il a déjà parcouru $\frac{2}{5}$ de la distance, ce qui représente 25km. Quelle est la distance totale de la course?

Primaire 6^e
Régulier

Les élèves du 2e cycle participent à des activités sportives. Ils ont le choix entre le hockey, l'escalade ou la natation. On sait que :

- deux fois plus d'élèves ont choisi le hockey que ceux qui ont choisi l'escalade;
- trois fois plus d'élèves ont choisi la natation que ceux qui ont choisi l'escalade;
- des élèves ont choisi la natation.

Primaire 5^e
Régulier